I: For each of the following questions, fill the blank space with the most simplified answers. (each question worth 2 points)

If 2 is the period of $f(x)$ and n is any integer, then the period of $f(x+2n)$ is ________

Assume $f(x)$ is a periodic function with period $2L$, and has a Fourier series on the interval $-L \le x \le L$, then its **Fourier series expansion formula** is

____

The **domain** of the function $f(x,y,z) = x\ln\left(\frac{1}{2y-z}\right)$ is ________

$\lim_{(x,y)\to(0,0)} \left(\frac{-y+x}{y^2-x^2}\right) =$ ________

5. Find the value of **c** such that the function $f(x,y)=\begin{cases} \dfrac{e^{x^2+y^2}-1}{x^2+y^2} & for \quad (x,y)\neq(0,0) \\ c, & for \quad (x,y)=(0,0) \end{cases}$ is **continuous** at $(0,0)$. **Answer:** c = ________

6. Given $z = x^2\ln y$, then $\frac{\partial^2 z}{\partial y \partial x}$ at (2,2) is ________

7. Let $f(x,y) = xe^{xy}$, $x = 3t$, and $y = 1$.
   i. $\frac{df}{dt} =$ ________
   ii. The equation of the tangent plane to the graph of f at a point (0,1,0) is ________

8. For $y^2(x^2 - 1) = zx^2$, by implicit differentiation $\frac{\partial z}{\partial y}$ at (1,1,0) = ________

9. If $R = \{(x,y): 0 \le x \le y, 0 \le y \le 1\}$, then $\iint_R e^{y^3}\, dA =$ ________

10. $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta\,\theta =$ ________

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TECHNOLOGY UNIVERSITY

DEPARTMENT

Examination

PART II: Workout the following problems by showing all the necessary steps. (30%)

1. Given $f(x) = \begin{cases} 0, & -5 < x < 0 \\ 1, & 0 < x < 5 \end{cases}$; $f(x+10) = f(x)$, then find the **Fourier series** of $f(x)$.

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5. Evaluate the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, \boldsymbol{dydx}$ by using **polar coordinates**.

4. By using **Lagrange's multipliers** method, find the **extreme values** of $f(x, y) = xy$ on the ellipse $2x^2 + y^2 = 1$.

2. The temperature at a point (x, y) on a metal plate in the xy plane is given by

$$T(x,y)=\frac{xy}{2+x+y}$$. Then find

a. all first and second order **partial derivatives** of T at (1,1).
b. the **gradient** vector, ∇T at the point $(1,1)$.
c. the **directional derivative** of T at $p(1,1)$ in the direction of the vector $v = i + j$.
d. the **maximum** value of the directional derivative at the point $p(1,1)$.